Compatible Taper and Volume Models for Atlantic White Cedar (*Chamaecyparis thyoides* L.) in Eastern North Carolina

A Thesis for the Degree
of Master of Science

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Presentation Outline

- Taper, and Volume Modeling
- Measurement sites and data collection methods
- Examination of existing taper model forms
- Development and quality determination of new taper and volume model forms
- Examination of regional effect on AWC taper curvature

Compatible Taper and Volume Models

"If tree profile can be accurately described, then volume for any merchantability limit or segment can be computed."
– Avery & Burkhart, *Forest Measurements*, 2002

• Variability of stem shape between species requires unique models to be developed

• Model forms developed for one species can be used to model another species with some possible bias and error

• No models specifically developed for AWC currently exist

Taper Modeling

Pre-computers –

Segmented/Switching: combination of three geometric shapes:

neiloid, frustum of a paraboloid, and cone

Post-computers:

Variable or Polynomial/Trigonometric: continuous curve typically a function of diameter at breast height (*D*), total height (*H*), and height (*h*) at which diameter (*d*) is predicted

Variable – $d(D,H,h) = f(D,H,h)^{g(D,H,h)}$

Polynomial – $d(D,H,h) = f(D,H,h) + g(D,H,h)$

Volume Modeling

Sectional Volumes

- Divide a stem in sections of known length (height)
- Requires diameters to be known for every section

$$\text{Volume}_{\text{Smalian's}} = \frac{(A_1 + A_2)}{2} L$$

Simple Linear Regression Models

- Single, Multiple, and combined variable model forms
- One volume estimate per stem

Single Entry – $V = \beta_0 + \beta_1 D^2$

Combined Variable – Spurr (1952) $V = \beta_0 + \beta_1 D^2 H$

Compatible Models

- Taper model form allows integration
- Volume model found by integrating cross sectional area
- Allows volume to be estimated to any height or diameter

Compatible Taper Model

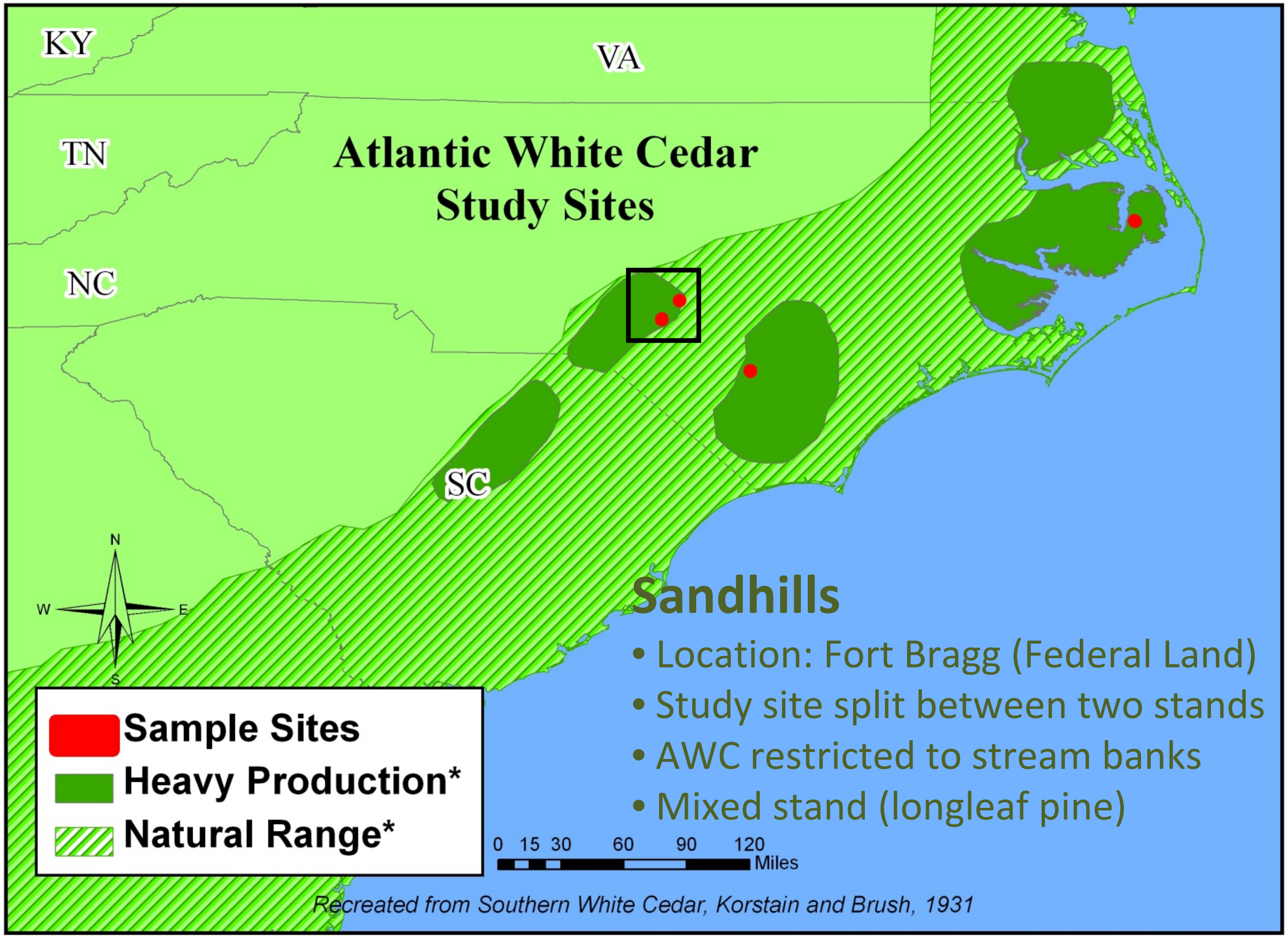
$$d(D,H,h) = f(D,H,h) + g(D,H,h)$$

Compatible Volume Model

$$V = \int \frac{\pi[d(D,H,h)]^2}{4} dh$$

MJC1

Study Sites



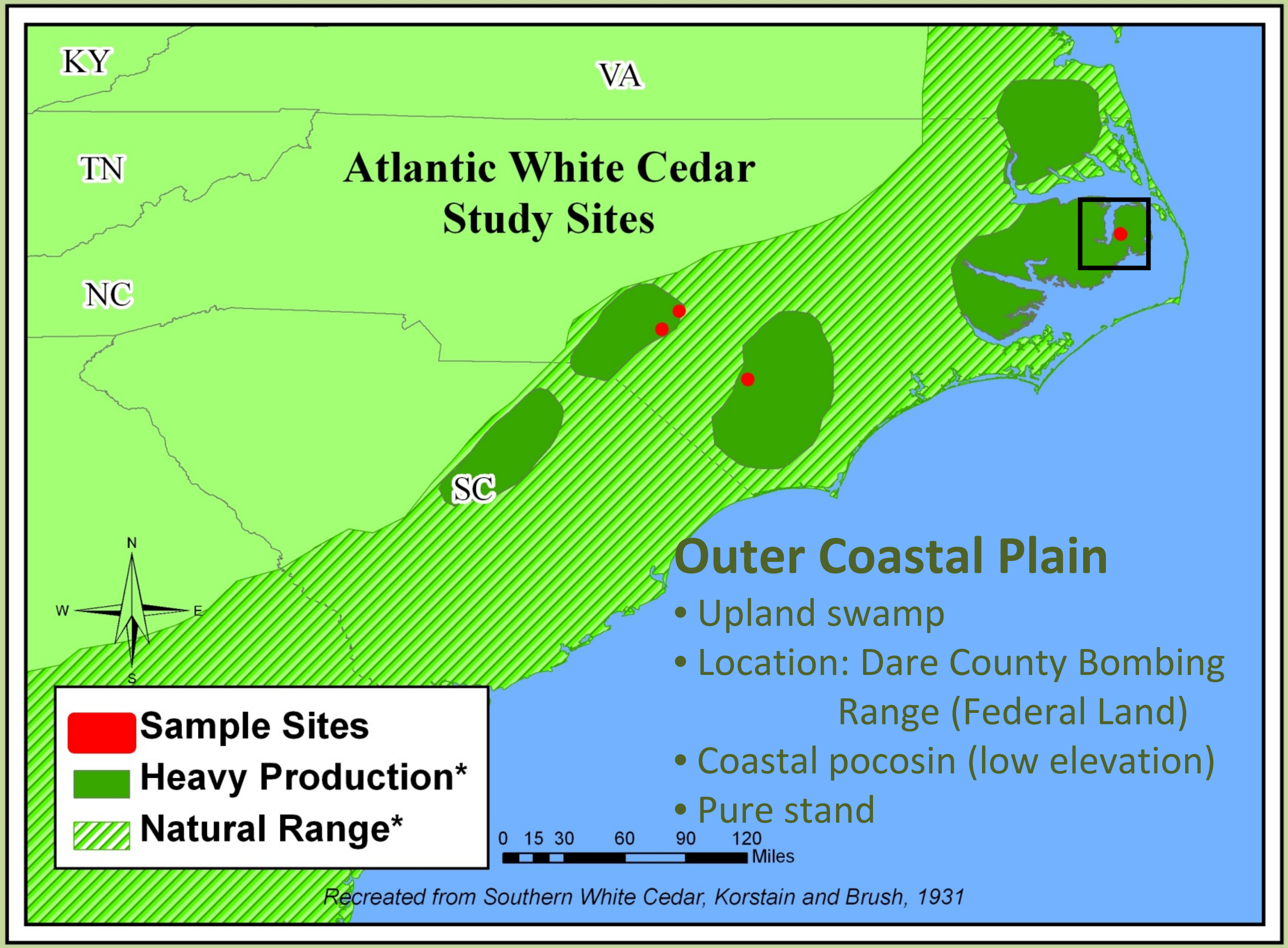
Sandhills

- Location: Fort Bragg (Federal Land)
- Study site split between two stands
- AWC restricted to stream banks
- Mixed stand (longleaf pine)

Slide 7

MJC1	Do I need these slides with the maps? Could I condense study sites to a table or remove completely? Matthew Cuneo, 6/8/2009

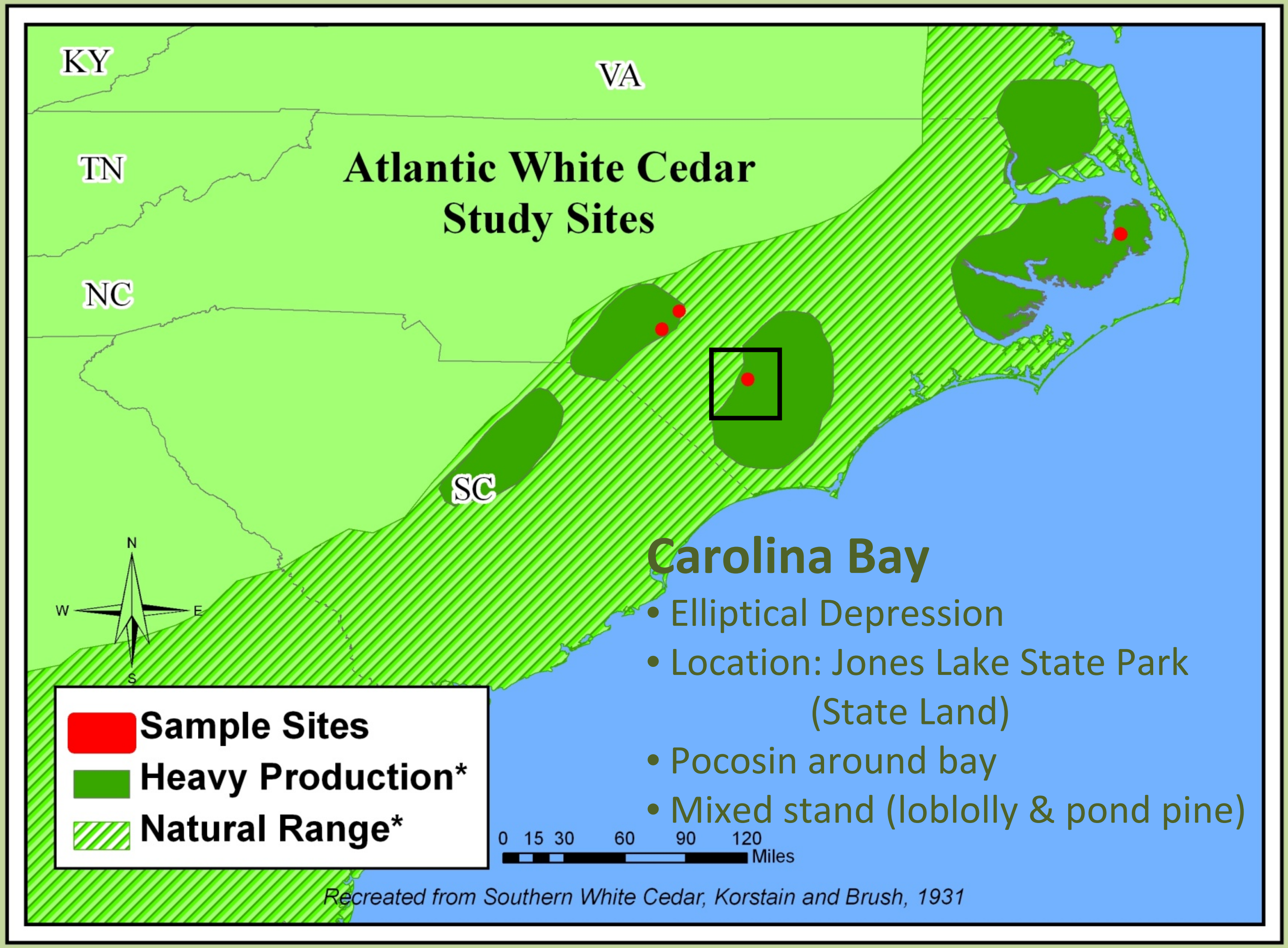
Study Sites



Outer Coastal Plain

- Upland swamp
- Location: Dare County Bombing Range (Federal Land)
- Coastal pocosin (low elevation)
- Pure stand

Study Sites



Carolina Bay

- Elliptical Depression
- Location: Jones Lake State Park (State Land)
- Pocosin around bay
- Mixed stand (loblolly & pond pine)

Measurement Tree Selection

- Dominant or Co-dominant
- Minimum 50% AWC overstory based on visual estimation
- Good form: No forking, sweeps, leaning, etc.
- Available line of site

Equipment

Diamond Measurements

Heights ≤ 6 ft

- Calipers

Heights > 6 ft (~5 ft interval)

Laser Technology Inc.

- Criterion RD 1000
    - Raw Inclination
    - Height / Diameter
        - ± 0.25” accuracy
- Laser Range Finder
    - Impulse
    - TruPulse

www.lasertech.com

Data Summary Means

Property	Statistic	Carolina Bays	Outer Coastal Plain	Sandhills	All Regions
Stems Sampled	Quantity	70	70	70	210
Measured Diameters	Quantity	**1050[a]**	**873[b]**	**884[b]**	2807
Basal Area per acre(ft^2/ac)	Mean	171	136[b]	126[b]	144
	SE	4.06	4.65	4.97	2.95
AWC Basal Area per acre (ft^2/ac)	Mean	104[c]	105[c]	62	90
	SE	4.19	5.17	4.35	2.99
Percent AWC	Mean	60.4	77.5	47.3	61.7
	SE	1.67	2.55	2.24	1.52

Means with a common letter are not significantly different at an alpha level of 0.05

Data Summary Means

Property	Statistic	Carolina Bays	Outer Coastal Plain	Sandhills	All Regions
Age at breast height (yrs)	Mean	**64[a]**	**55[b]**	**56[b]**	58
	SE	1.4	1.5	2.5	1.1
Diameter at Breast Height (in)	Mean	**13.2[a]**	**10.4[b]**	**11.2[b]**	11.6
	SE	0.42	0.38	0.44	0.25
Total Height (ft)	Mean	**82.5[a]**	**60.1[b]**	**62.2[b]**	68.2
	SE	1.25	0.69	1.15	0.93
Height to Live Crown (ft)	Mean	54.3	40.2	30.8	41.9
	SE	1.05	0.64	0.98	0.85
Radial Growth (in/10 yrs)	Mean	**0.57[a]**	**0.66[b]**	**0.73[b]**	0.69
	SE	0.03	0.03	0.03	0.02

Means with a common letter are not significantly different at an alpha level of 0.05

Fitting Existing Taper Model Forms

- 18 taper models selected
    - 17 polynomial, 1 polynomial and trigonometric
- Mixed modeling techniques used to fit each model

Model	-2 LL	AIC*	AICC	BIC
Demaerschalk (1972)	-8094	**-8090**	-8090	-8079
Thomas and Parresol (1991)	-5150	-5132	-5132	-5102
Demaerschalk (1972)*	-3764	-3760	-3760	-3748

* Smaller is a better fit
[R] Random Effect
[NS] Not significant at an alpha level of 0.05

Additional Definitions

Relative Diameter –

$$\frac{diameter}{Diameter\ at\ breast\ height}$$

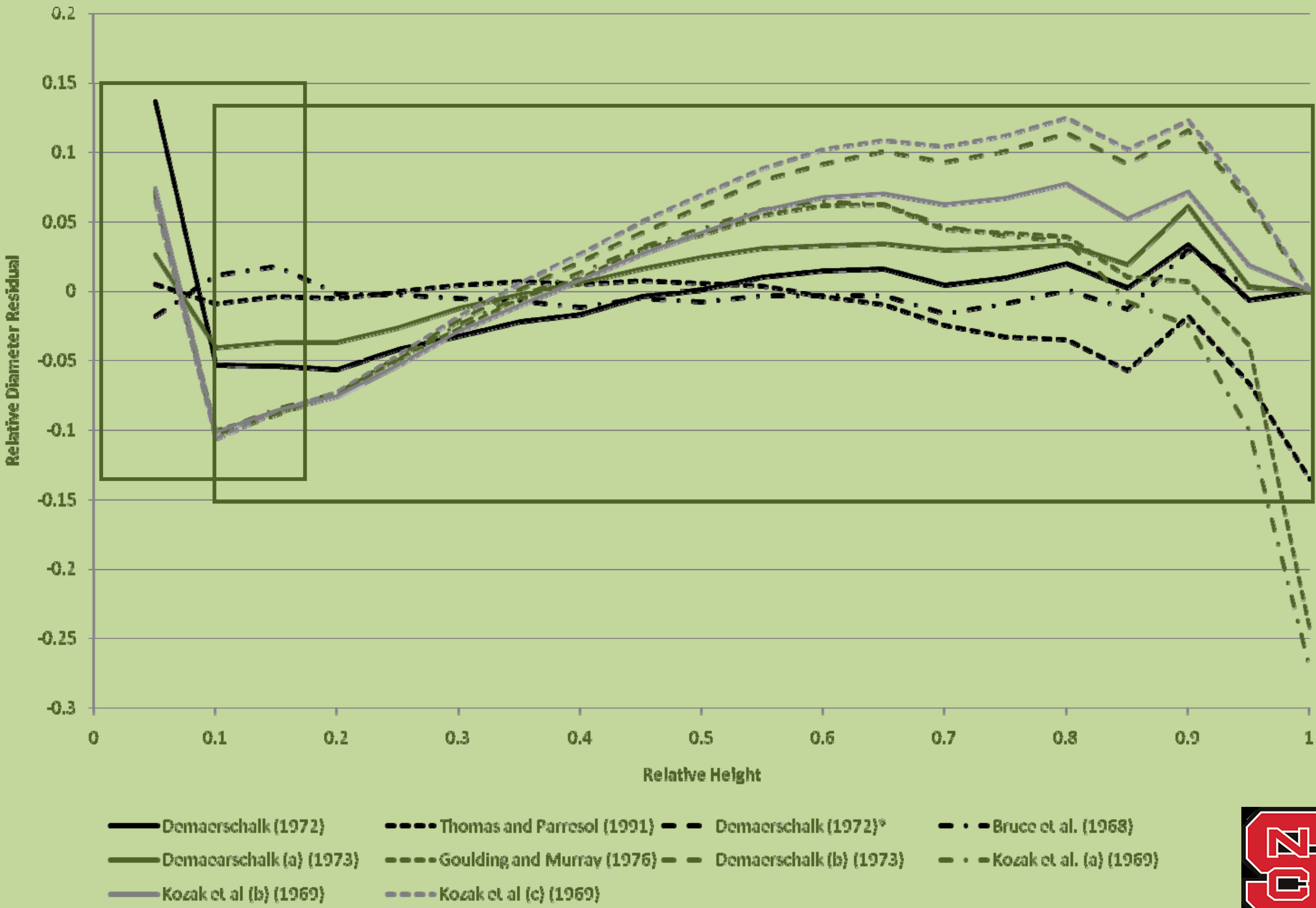
Relative Height –

$$\frac{height}{Total\ height}$$

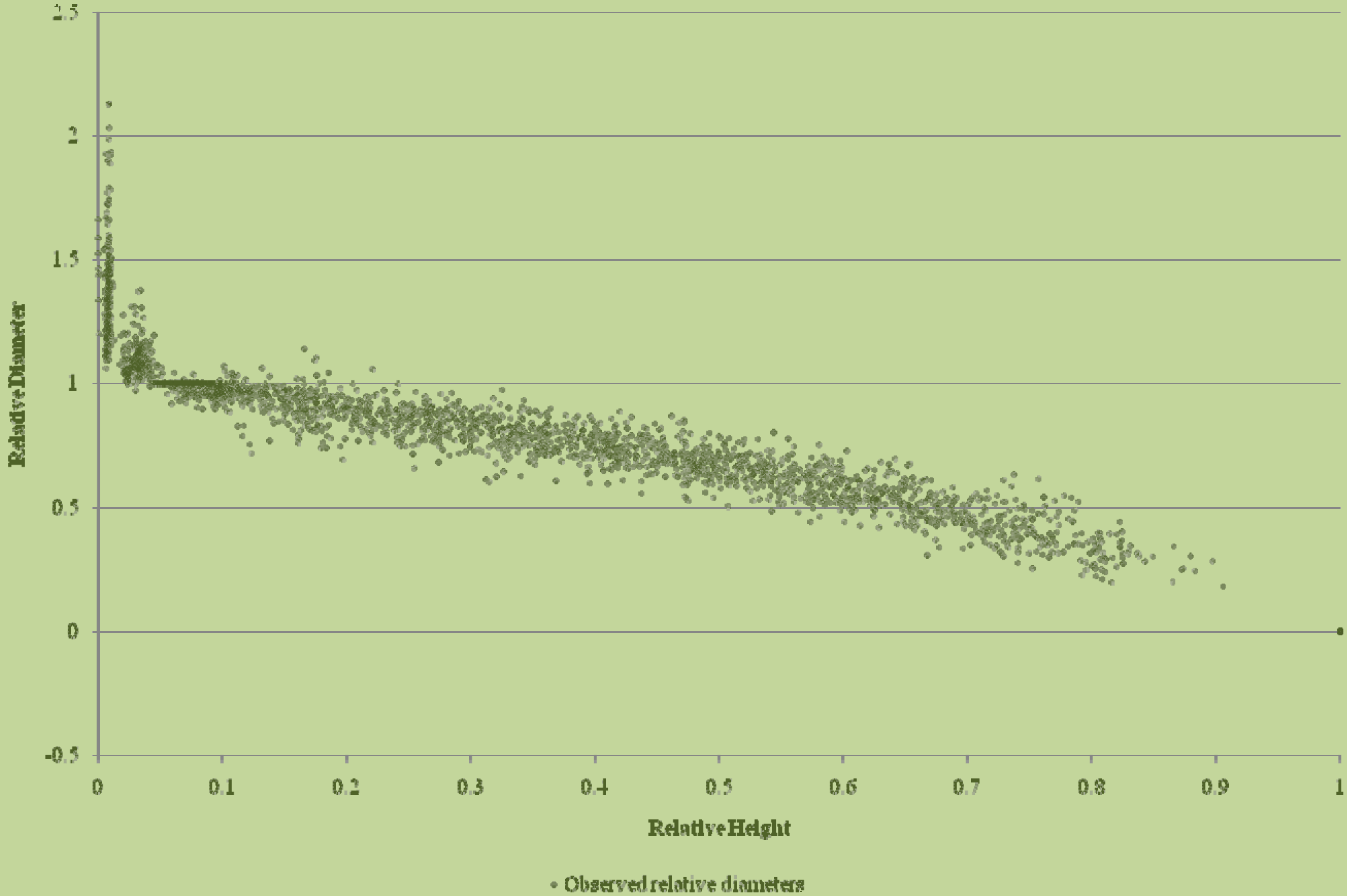
Relative Diameter Residual –

$$Observed_{Rel\ Dia} - Predicted_{Rel\ Dia}$$

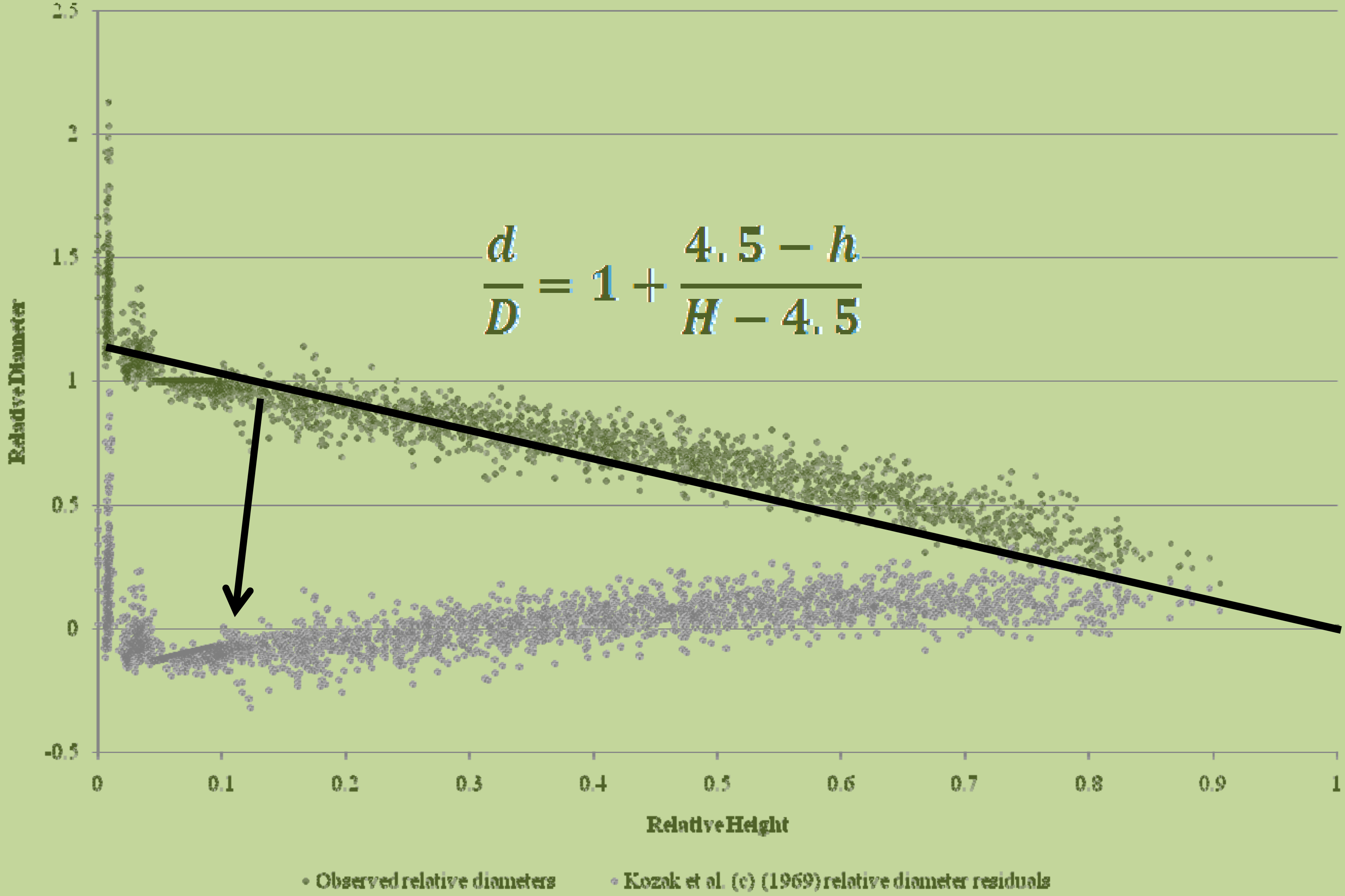
Mean Relative Diameter Residuals



Observed Relative Diameters

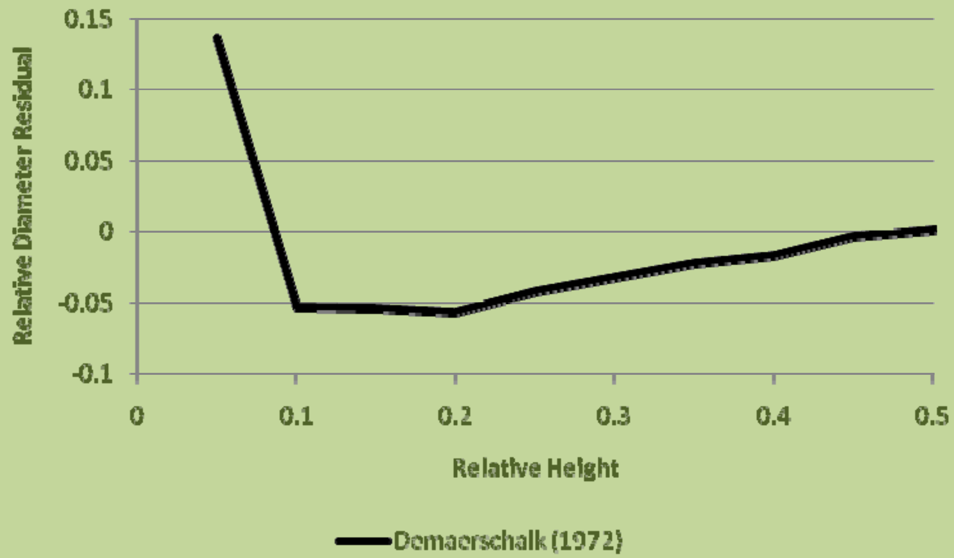


Comparison of Graphs

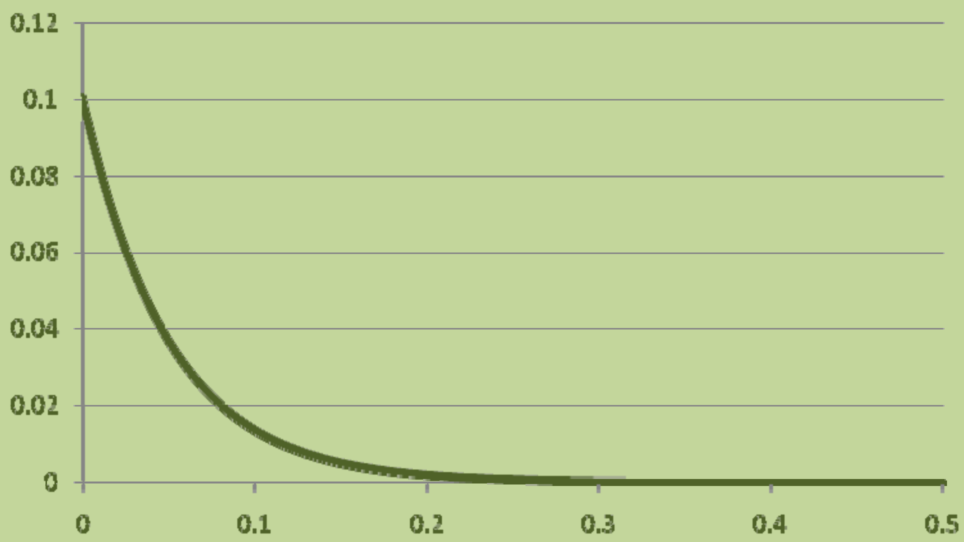


$$\frac{d}{D} = 1 + \frac{4.5 - h}{H - 4.5}$$

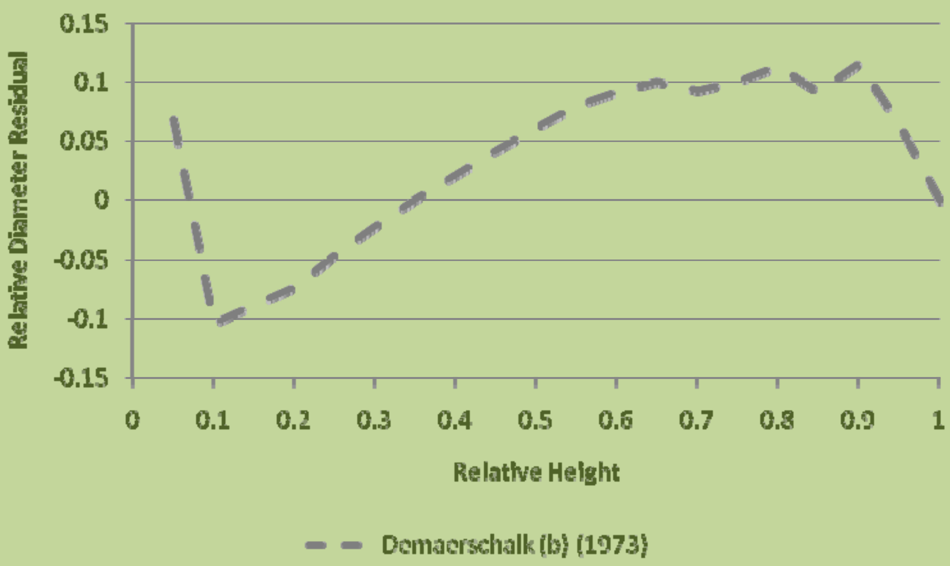
Exponential Decay Curve



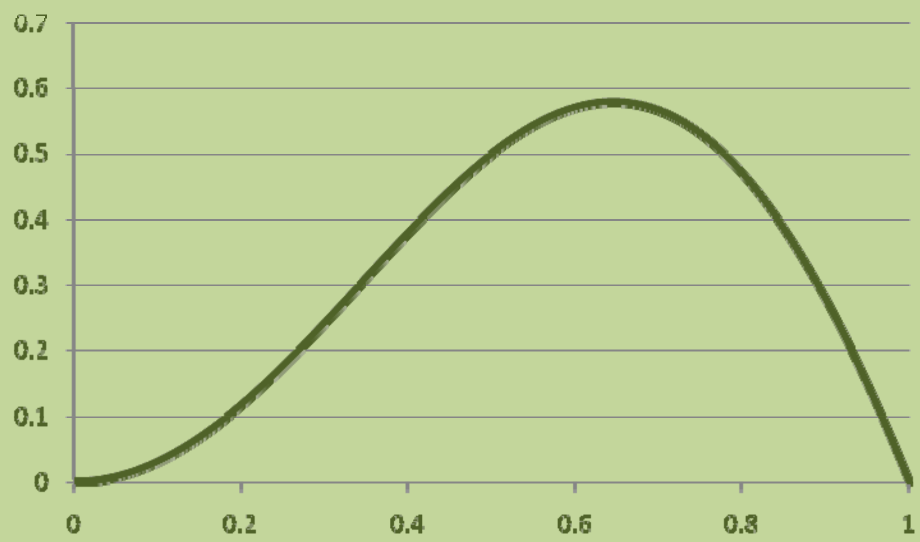
$$E = \frac{E_0}{e^{\lambda x}}$$



Sine Function



$$S = x \sin(\pi x)$$



L.E.S.

Linear Function (L)

$$\frac{d}{D} = \beta_1\left(1 + \frac{4.5 - h}{H - 4.5}\right)$$

Exponential Decay Curve (E)

$$E = \frac{\beta_2}{e^{\beta_3 h}}$$

Sine Function (S)

$$S = \beta_4 \frac{h}{H} \sin\left(\pi \frac{h}{H}\right)$$

L.E.S.

$$\frac{d}{D} = \beta_1\left(1 + \frac{4.5 - h}{H - 4.5}\right) + \frac{\beta_2}{e^{\beta_3 h}} + \beta_4 \frac{h}{H} \sin\left(\pi \frac{h}{H}\right) + \varepsilon$$

Fitting L.E.S. to AWC data

Components	Model	β_1	β_2	β_3	β_4	-2 LL	AIC	AICC	BIC
L.	$\frac{d}{D} = \beta_1\left(1 + \frac{4.5 - h}{H - 4.5}\right)$	1.107	.	.	.	-3930	-3926	-3926	-3914
L.E.	$\frac{d}{D} = \beta_1\left(1 + \frac{4.5 - h}{H - 4.5}\right) + \frac{\beta_2}{e^{\beta_3 h}}$	1.07[R]	0.60[R]	2.00	.	-5041	-5029	-5029	-5009
L.S.	$\frac{d}{D} = \beta_1\left(1 + \frac{4.5 - h}{H - 4.5}\right) + \beta_4 \frac{h}{H} \sin\left(\pi \frac{h}{H}\right)$	1.066	.	.	0.193	-4565	-4561	-4561	-4549
L.E.S.	$\frac{d}{D} = \beta_1\left(1 + \frac{4.5 - h}{H - 4.5}\right) + \frac{\beta_2}{e^{\beta_3 h}} + \beta_4 \frac{h}{H} \sin\left(\pi \frac{h}{H}\right)$	0.99[R]	0.52[R]	1.06	0.27[R]	-9002	-8986	-8986	-8959

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Best 3	
Demaerschalk (1972)	-8090
Thomas and Parresol (1991)	-5132
Demaerschalk (1972)*	-3760

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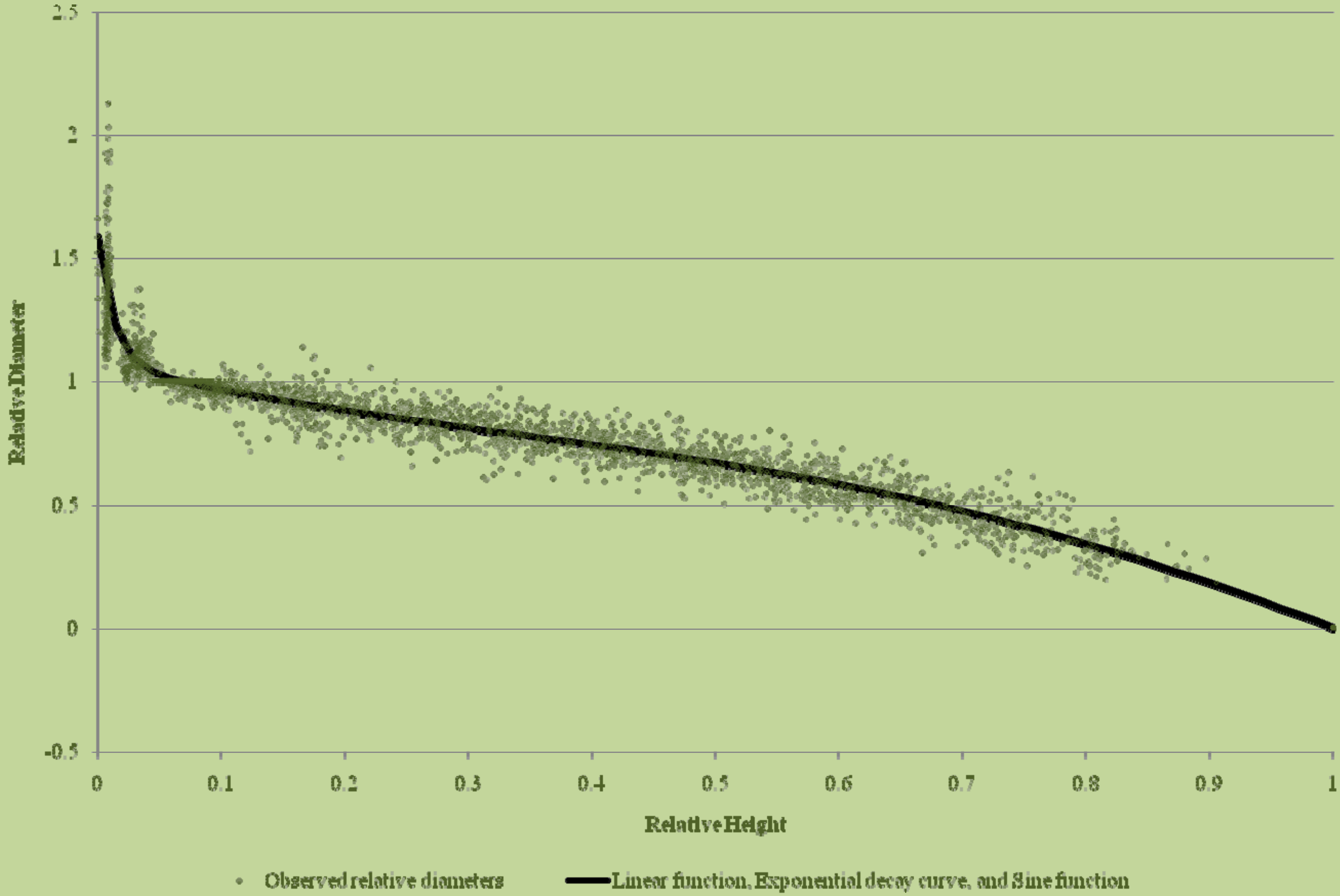
Observed Relative Diameters



Compatible Volume Model

Assume a circular cross section for all relative heights

$$V = \int A_h \, dh = \int \frac{\pi {d_h}^2}{4} \, dh$$

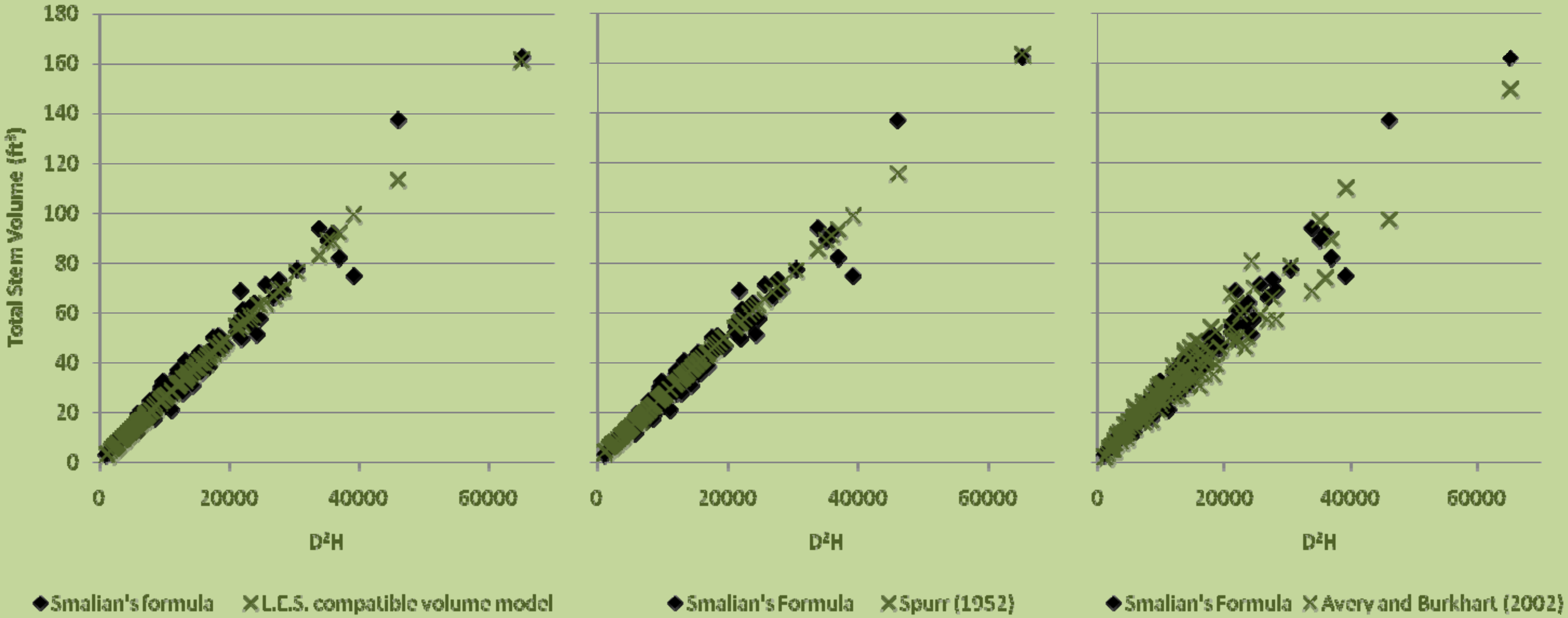
L.E.S. Volume Model $$V_{h_2 - h_1} = V_{h_2} - V_{h_1}$$

Where:

$$\begin{aligned} V_h = \frac{\pi}{576} D^2 \Bigg[& {\beta_1}^2 h + \frac{2{\beta_1}^2}{H - 4.5}\left(4.5h - \frac{h^2}{2}\right) - \frac{2\beta_1\beta_2}{\beta_3 e^{\beta_3 h}} + \left(\frac{2\beta_1\beta_4}{\pi^2} + \frac{9\beta_1\beta_4}{\pi^2(H - 4.5)}\right)\left[H \sin\left(\pi\frac{h}{H}\right) - \pi h \cos\left(\pi\frac{h}{H}\right)\right] \\ & + \frac{{\beta_1}^2}{(H - 4.5)^2}\left(20.25h - 4.5h^2 + \frac{h^3}{3}\right) + 2\beta_1\left[\frac{-\beta_2(4.5 - h)}{(H - 4.5)\beta_3 e^{\beta_3 h}} + \frac{\beta_2}{(H - 4.5){\beta_3}^2 e^{\beta_3 h}}\right] \\ & + \frac{2\beta_1\beta_4}{\pi^3(H - 4.5)}\left[\cos\left(\pi\frac{h}{H}\right)(\pi^2 h^2 - 2H^2) - 2\pi h H \sin\left(\pi\frac{h}{H}\right)\right] - \frac{{\beta_2}^2}{2\beta_3 e^{2\beta_3 h}} \\ & + \frac{-2\beta_4\beta_2 e^{-\beta_3 h}}{\left({\beta_3}^2 H^2 + \pi^2\right)^2}\left[H \sin\left(\pi\frac{h}{H}\right)\left({\beta_3}^3 H^2 h + \pi^2 h \beta_3 - \pi^2 + {\beta_3}^2 H^2\right)\right. \\ & \left. + \pi \cos\left(\pi\frac{h}{H}\right)\left({\beta_3}^2 H^2 h + \pi^2 h + 2\beta_3 H^2\right)\right] \\ & + \frac{{\beta_4}^2}{24\pi^3 H^2}\left[4\pi^3 h^3 - 6\pi H^2 h \cos\left(2\pi\frac{h}{H}\right) + 3H \sin\left(2\pi\frac{h}{H}\right)(H^2 - 2\pi^2 h^2)\right] \Bigg] \end{aligned}$$

Quality of Volume Model

Original Author	Model	β_0	β_1	R^2
L.E.S. Compatible Volume Model	$V = \int \frac{\pi (D * L.E.S.)^2}{4} dh$	.	.	0.9701
Spurr (1952)	$V = \beta_0 + \beta_1 D^2 H$	0.935	0.002	0.9740
Single Entry	$V = \beta_0 + \beta_1 D^2$	-3.804	0.215	0.8974



Improved Regional Differences Modeling

Fitting each region individually allows:

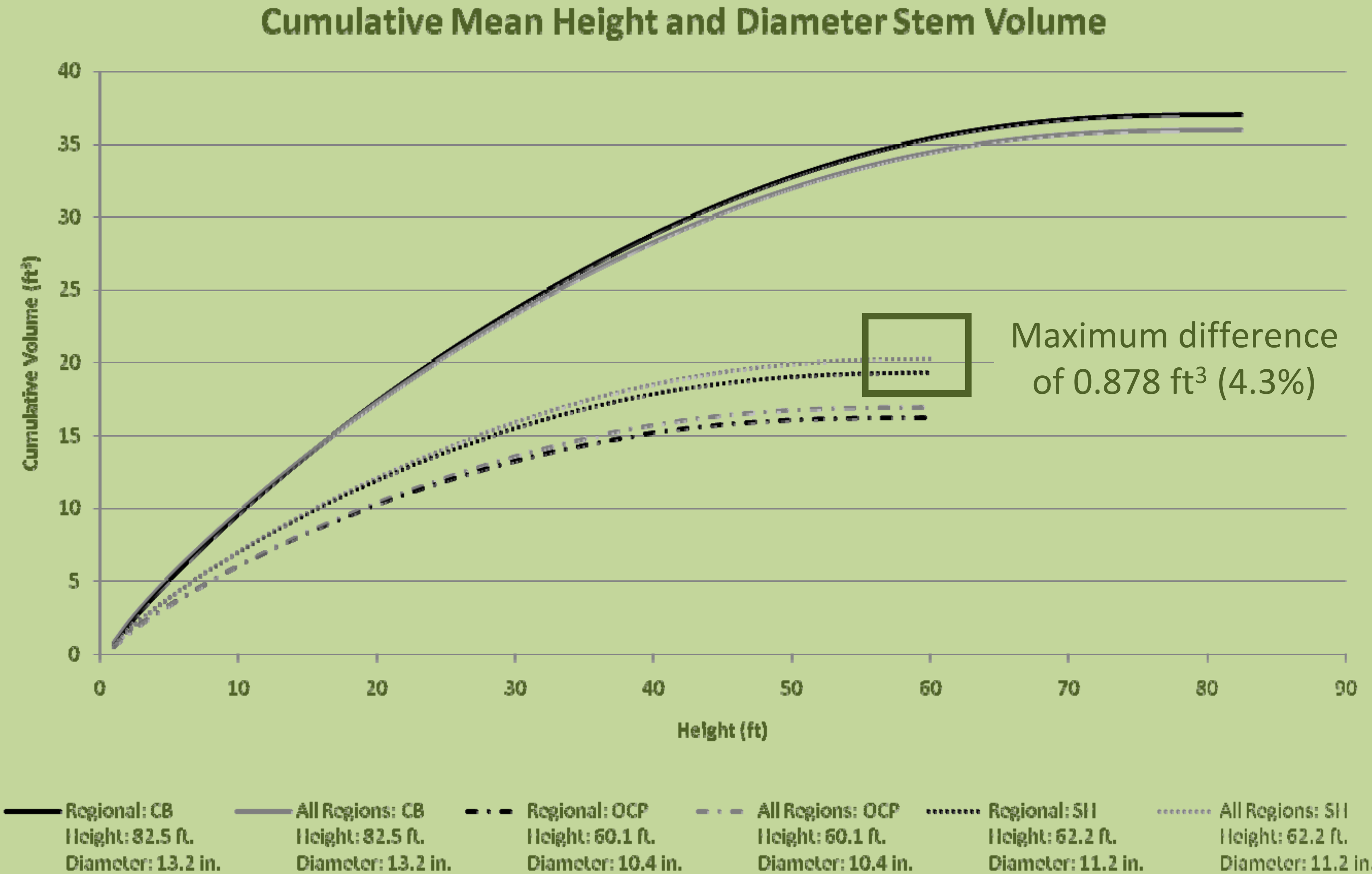
- Difference taper curvature
- Relative Diameter at Total Height constricted to zero

Region	Model Parameters	-2 LL	AIC	AICC	BIC	β_1	β_2	β_3	β_4
CB	All Regions*	-3334	**-3332**	-3332	-3330	0.994^R	0.523^R	1.058	0.275^R
	Regional	-3384	**-3368**	-3368	-3350	1.007^R	0.378^R	1.222	0.297^R
OCP	All Regions*	-2841	**-2839**	-2839	-2837	0.994^R	0.523^R	1.058	0.275^R
	Regional	-2886	**-2870**	-2870	-2852	0.992^R	0.707^R	1.118	0.218^R
SH	All Regions*	-2836	**-2834**	-2834	-2832	0.994^R	0.523^R	1.058	0.275^R
	Regional	-2866	**-2850**	-2850	-2832	0.982^R	0.500^R	0.895	0.312^R

^{R} Random Effect Term

*Model parameters found when fitting the L.E.S. taper model to the combined regions data set.

Regional Effect on Volume Modeling



Summary of Conclusions

- L.E.S. taper model fits AWC better than any of the examined model forms

- The compatible volume model does not provide any improvement of total stem volume estimation over Spurr (1952)

- The compatible volume model does allow for sectional volume estimates between any two heights or diameters

- The effect of region on AWC taper is statistically significant and appears as differences in taper curvature.

- No practical improvement when using a regional taper model

- New models available for AWC forestland managers to calculate total volume and merchantable volume estimates

Questions?